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ON FRONTS OF STRONG AND WEAK DISCONTINUITIES IN SOLUTIONS OF THE EQUATIONS OF DIFFERENT-MODULUS ELASTICITY THEORY*

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The fronts in a different-modulus elastic body on which a change in the elastic properties occurs are classified. Fronts representing strong discontinuities (shocks) of low intensity as well as their corresponding simple waves are considered. The existence of weak discontinuities on which the conditions of continuity and the condition giving the change in the elastic properties do not yield a complete system of relationships on the front is proved. In this case an additional relationship is postulated.

Models of different-modulus elastic bodies, i.e., bodies whose elastic properties change in a discontinuous manner, are proposed in /1-5/. Experimental investigations underlying the models are listed in /4-7/. The different-moduli phenomenon is often explained by the presence of shallow cracks, pores in the body. Different-moduli in fluids can be generated by phase transitions. Simple waves and certain other solutions for a barotropic medium in which the pressure dependence on the density has a break corresponding to fluid boiling have been obtained /8/. Solutions of the equations of motion of a different-modulus elastic medium corresponding to longitudinal waves were investigated /9-11/. The formulation and solution of certain problems for a two-parameter medium, a fluid with equilibrium phase transitions, have been given by Galin**. (**G.Ya. Galin, Phase transformation waves. Proceedings of the International Conference on "Modern Mathematical Problems of Mechanics and their Applications", Moscow, 1987.) Solutions of a number of static problems of elasticity theory are presented in /6/.

Below, plane waves that are described by a complete system of elasticity theory equations, that are a hyperbolic system of seventh-order equations, are examined in an arbitrary elastic body.

1. *Characteristic velocity discontinuities.* Plane waves in an elastic medium can be described by the equations /12, 13/.

$$\begin{gathered}\rho\frac{\partial v_i}{\partial t}=\frac{\partial}{\partial x}\left(\frac{\partial\Phi}{\partial u_i}\right)+F_i(x,t)\equiv A_{ij}\frac{\partial u_j}{\partial x}+B_i\frac{\partial S}{\partial x}+C_i+F_i\\ \frac{\partial u_i}{\partial t}=\frac{\partial v_i}{\partial x};\quad \frac{\partial S}{\partial t}=0\\ \Phi=\Phi(u_iS,x);\quad A_{ij}=\frac{\partial^2\Phi}{\partial u_i\partial u_j};\quad B_i=\frac{\partial^2\Phi}{\partial S\partial u_i}\\ C_i=\frac{\partial^2\Phi}{\partial x\partial u_i};\quad u_i=\frac{\partial w_i}{\partial x};\quad v_i=\frac{\partial w_i}{\partial t}\end{gathered}\tag{1.1}$$

Here $x_1, x_2, x_3 = x$ are Cartesian coordinates whose values for points of the medium at the initial time are taken as Lagrange variables, w_i are displacement vector components in this coordinate system, ρ is the density of the initial state, and S is the entropy. The quantities u_i, v_i, S are considered to be functions of x and t, while, $\Phi = \rho U$ is the internal energy of unit initial volume of the medium considered as a function of the gradient tensor components of the displacements u_i varying in the wave, the entropy S and possibly the Lagrange coordinate x.

The matrix A_{ij} is constant in the classical theory of a linearly elastic body and $B_i = 0$, $C_i = 0$. It will be assumed here that the second derivatives A_{ij} of the elastic

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potential Φ undergo a discontinuity while the potential itself and its first derivatives with respect to u_j are continuous. The dependence of Φ on S and x is considered to be fairly smooth. We consider the discontinuity of A_{ij} in the space of the variables u_i located on a certain surface $\psi(u_i, S, x) = 0$ defined by giving Φ as a function of its arguments. We call this surface the separation surface (SS). Its position in the space u_i depends on x and S as on the parameters.

Therefore, the SS is a second-order surface of weak discontinuity of the potential Φ considered as a function of u_i. As we known, only the second derivative of Φ with respect to the normal undergoes a discontinuity on such a surface. The SS divides the space u_i into two parts which we denote by V^* and V^{**} and we give all quantities one or two asterisks if it is necessary to distinguish to what part of the **domain** in space the quantity refers.

The characteristic velocities of system (1.1) are connected with the eigenvalues $\lambda^{(k)}$ of the matrix A_{ij} by the relationships $c^{(k)} = (\lambda^{(k)}/\rho)^{1/2}(k = 1, 2, 3)$. Both $\lambda^{(k)}$ and $c^{(k)}$ are discontinuous together with A_{ij} when passing through the SS. We will investigate these changes.

We select a certain point u_i° on the SS for certain fixed values of the variables S and x. We introduce the new variable u_i' related to u_i by an orthogonal transformation with constant coefficients. We direct the u_1' axis parallel to the normal to the SS at the point u_i°. We direct the axes u_2' and u_3' so that they are orthogonal to the u_1' axis so that $\partial^2\Phi/\partial u_2'\partial u_3' = 0$. We define new quantities A_{ij}, B_i, C_i by the same derivatives as in (1.1) but in the new variable u_i'. This is obviously equivalent to a tensor transformation of the original quantities. Let us also transform v_i and F_i to the new axis. The transformations described above retain (1.1) as valid, since the left and right sides of these equations are transformed according to an identical vector law. We shall later use the new variables **and** omit the primes.

In the new coordinate system the equation $\psi = 0$ is written as $u_1 = u_1^\circ$ in the neighbourhood of the point u_i° to within higher-order infinitesimals, and only A_{11} out of all the components of the matrix A_{ij} is discontinuous at the point u_i°: $A_{11}^{**} - A_{11}^* \neq 0$. To fix our ideas, we will assume $A_{11}^{**} > A_{11}^*$.

We write the characeristic equation to find the proper roots, while taking into account that $A_{23} = 0$, in the new coordinate system $(A_{22} - \lambda)\cdot(A_{33} - \lambda)\, P(\lambda) = 0$

$$P(\lambda) \equiv A_{12}^2/(\lambda - A_{22}) + A_{13}^2/(\lambda - A_{33}) + A_{11} - \lambda \tag{1.2}$$

If

$$A_{12} \neq 0, \quad A_{13} \neq 0, \quad A_{22} \neq A_{33} \tag{1.3}$$

then the values $\lambda = A_{22}, \lambda = A_{33}$ are not roots of (1.2). We will henceforth assume that $\lambda \neq A_{22}, \lambda \neq A_{33}$. To be specific we assume $A_{22} < A_{33}$ everywhere.

A graph of the function $P(\lambda)$ is shown in Fig.1: $\lambda^{(k)}$, $(k = 1, 2, 3)$ are the roots of the equation $P(\lambda) = 0$, they shift to the right as A_{11} increases and to the left as it decreases. Therefore, when satisfying conditions (1.3), the eigenvalues and characteristic velocities on different sides of the SS satisfy the inequalities

$$\begin{gathered}\lambda_*^{(1)} < \lambda_{**}^{(1)} < \lambda_*^{(2)} < \lambda_{**}^{(2)} < \lambda_*^{(3)} < \lambda_{**}^{(3)} \\ c_*^{(1)} < c_{**}^{(1)} < c_*^{(2)} < c_{**}^{(2)} < c_*^{(3)} < c_{**}^{(3)}\end{gathered} \tag{1.4}$$

It is interesting to note that inequalities analogous to (1.4) also hold during the elasticity-plasticity transition /14/, but one result is not a consequence of the other.

2. *Simple waves.* Let us examine the solution of (1.1) for $F_i = 0$, $S = \text{const}$, $\partial\Phi/\partial x = 0$ in the form $u_i = u_i(\theta(x, t))$, $v_i = v_i(\theta(x, t))$, where $\theta(x, t) \neq \text{const}$ is a certain function. Such solutions are called simple waves. Substitution of the above-mentioned kinds of solutions into (1.1) yields

$$(A_{ij} - \rho c^2\delta_{ij})\frac{du_j}{d\theta} = 0, \quad \frac{\partial\theta}{\partial t} + c\frac{\partial\theta}{\partial x} = 0 \tag{2.1}$$

It is hence seen that $\rho c^2 = \lambda$ is one of the eigenvalues of the matrix A_{ij}, while the derivatives $du_i/d\theta$ are proportional to the appropriate eigenvector g_i of the matrix A_{ij}. A simple wave obviously corresponds to each eigenvector of the matrix A_{ij}.

We will examine a small neighbourhood of a certain point u_i° of the interfacial surface and assume that the matrix A'_{ij} takes the constant values A_{ij}^{**} in the domain V^{**} and A_{ij}^* in the domain V^*. The characteristic velocities will also be picewise-constant. Under such assumptions we neglect the process of slow wave deformation associated with the variability of c in each of the domains V^{**} and V^* leaving the wave deformation associated just with the jump c.

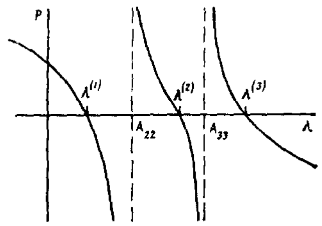

Fig.1

The solution in each of the domains V^* or V^{**} is constructed as in the linear case, where the u_i vary along a line parallel to the eigenvector in this domain while the solution contains the factor $f(x-ct)$, where f is an arbitrary function (that can contain the discontinuity). This means that in a simple wave the u_i vary along a broken line with the break point on the SS (in certain special case it can turn out that there is no breakpoint).

We will examine the change in the eigenvectors on passing through the SS. It can be seen that when conditions (1.3) are satisfied, all the eigenvectors have a component g_1 different from zero that is normal to the interfacial surface. This follows from the last two equations for the eigenvector components under the condition that (see Sect.1) $A_{23}=0$, $\lambda \neq A_{22}$, $\lambda \neq A_{33}$, that is

$$A_{12}g_1 + (A_{22}-\lambda)g_2 = 0, \quad A_{13}g_1 + (A_{33}-\lambda)g_3 = 0 \tag{2.2}$$

According to Sect.1, a decrease in A_{11} leads to an increase in $|A_{22}-\lambda^{(1)}|$ and $|A_{33}-\lambda^{(1)}|$ and therefore, to a decrease in $g_2^{(1)}/g_1^{(1)}$ and $g_3^{(1)}/g_1^{(1)}$ so that the eigenvector deviates towards the normal to the SS. We similarly obtain that as A_{11} diminishes, the eigenvector $g_i^{(3)}$ approaches the SS in the direction to the tangent plane. The ratio $|g_2^{(2)}/g_1^{(2)}|$ increases for the eigenvector $g_i^{(2)}$ while $|g_3^{(2)}/g_1^{(2)}|$ decreases.

When one or more of conditions (1.3) are violated, one or two eigenvectors become parallel to the tangent plane to the SS. This case will not be considered henceforth.

The conditions of non-reversal of simple waves are obviously included in the case being considered in that the leading part of the wave should correspond to the greater value of $A_{11}=A_{11}^{**}$ and the trailing part to the smaller value of $A_{11}=A_{11}^{*}$, i.e., a change in magnitude occurs in the non-reversing wave so that the point $u_i(x,t)$ goes from V^{**} to V^* for $x=\text{const}$ as t increases. In physical space, the leading and trailing parts of the simple wave, in which $u_i \in V^{**}$ and $u_i' \in V^*$, respectively, travel without deformation with constant but different velocities c^{**} and c^*, while a domain of constant values $u_i=u_i^\circ$ corresponding to the point on the boundary $\psi(u_i)=0$ between V^{**} and V^* is formed between them and increases linearly with time. For the opposite direction of the change in the quantities in a simple wave, the trailing part of the wave overtakes the leading part and an ambiguity occurs that should be removed by the introduction of a discontinuity (shockwave) into the solution.

3. *Shockwaves.* Relationships expressing the conservation of momentum, energy and continuity of the displacements should be satisfied on the discontinuities

$$\rho W[v_i] = -[\partial\Phi/\partial u_i], \; [\Phi] = \{(\partial\Phi/\partial u_k)^- + {}^1/_2[\partial\Phi/\partial u_k]\}[u_k]$$
$$W[u_i] = -[v_i] \tag{3.1}$$

As is usual, here the square brackets denote jumps in the quantities in the brackets $[u_i]=u_i^+-u_i^-$, where the quantity directly in front of the discontinuity is denoted by a minus superscript and that behind the discontinuity by a plus superscript, and W is the velocity of the jump dx/dt.

Let us calculate the change in entropy in the discontinuity by assuming that the initial and final states are sufficiently close to the SS corresponding to the initial value of the entropy. Assuming the change in entropy in the wave to be small, we limit ourselves to a linear expansion of the potential Φ in the entropy at the point S^-

$$\Phi(u_i,S) = \rho T^-(S-S^-) + \Phi_1(u_i) \tag{3.2}$$

It then follows from (3.1) that

$$\rho T^-[S] = {}^1/_2[\partial\Phi_1/\partial u_i][u_i] + (\partial\Phi_1/\partial u_i)^-[u_i] - [\Phi_1] \tag{3.3}$$

We expand the right side of the last equality in series in the quantities $\Delta u_i^\pm$ in the neighbourhood of a certain point u_i° on the SS, where $\Delta u_i^\pm = u_i^\pm - u_i^\circ$ and we expand to second-order infinitesimals. Taking into account that only the second derivatives of Φ_1 are discontinuous, we obtain for $[S]$

$$\rho T^-[S] = \{A_{ij}^-\Delta u_j^- + {}^1/_2[A_{ij}u_j]\}[u_i] - {}^1/_2[A_{ij}u_iu_j] \tag{3.4}$$

In the case under consideration, when only A_{11} is discontinuous of all the expansion coefficients, all terms not containing A_{11} are cancelled from (3.4), we consequently have

$$\rho T^-[S] = {}^1/_2[A_{11}]\,|\Delta u_1^-|\,|\Delta u_1^+| \tag{3.5}$$

It is seen that only under the condition $A_{11}^+ - A_{11}^- \geqslant 0$ does the jump satisfy the requirement of a non-decrease in the entropy; in other words, only jumps with an increase in A_{11}, i.e., from V^* into V^{**} are possible.

We note that these deductions are in agreement with the simple wave behaviour considered above in the sense that shockwaves exist when "reversal" of the appropriate simple waves occurs and do not exist otherwise.

As is seen from (3.5), the change in entropy in the shock does not exceed the second order smallness but can be less or even vanish when either Δu_1^- or Δu_1^+ equals zero (in the latter case the initial and final states of the medium do not emerge outside the limits of the domain of linear behaviour).

4. *Evolutionary conditions.* The inequalities (1.4) enable us to list all the evolutionary fronts both with a continuous change in magnitude and those representing low-intensity discontinuities. In the latter case we shall also consider that the characteristic velocities in the states ahead of and behind the discontinuity agree, respectively, with $c_*^{(k)}$ and $c_{**}^{(k)}$.

As we know /15/, the evolutionary conditions in the general case are that the number of boundary conditions on a discontinuity should be one greater than the number of characteristics leaving the discontinuity. The velocity axis which the values $c_*^{(k)}$ and $c_{**}^{(k)}$ divide into intervals, is shown in Fig.2. If the velocity of the discontinuity W belongs to any of these intervals, then the number of boundary conditions on the discontinuity needed for evolutionarity can be indicated depending on whether the states before and behind the front correspond to the states marked with the single and double asterisks, or conversely. The number of boundary conditions is indicated in Fig.2 above and below the appropriate intervals of velocity change.

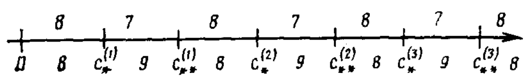

Fig.2

5. *The change in the quantities in the shocks, confirmation of the evolutionarity conditions.* Considering the shock to be weak and neglecting second-order infinitesimals (including the entropy change), we obtain from the first and third relationships in (3.1)

$$A_{ij}^+[u_j] = \alpha[u_i] - [A_{ij}]\Delta u_j^- \quad (\alpha = \rho W^2) \tag{5.1}$$

Taking into account that according to Sect.1 only A_{11} has a non-trivial discontinuity and $A_{23} = 0$, we obtain from (5.1)

$$[u_k] = A_{1k}(\alpha - A_{kk})^{-1}[u_1], \quad k = 2, 3; \quad P^+(\alpha)[u_1] = -[A_{11}]\Delta u_1^- \tag{5.2}$$

where the function P is determined by (1.2) and the plus superscript denotes that $A_{11} = A_{11}^+$. Obviously $P^+(\lambda_-^{(k)}) = [A_{11}] > 0$, and we obtain $\Delta u_1^+(\lambda_-^{(k)}) = 0$ from the last equality in (5.2). It follows from this same equality that an increase in Δu_1^+ occurs as $P^+(\alpha)$ diminishes, i.e., as α increases (Fig.1). For $\alpha = \lambda_+^{(k)}$ $P^+(\alpha)$ vanishes, and Δu_1^+ becomes infinite. Taking account of the first two equalities in (5.2), we obtain the related branch of the curve in the space u_i

$$u_i^+ = u_i^+(W), \quad c_-^{(k)} \leqslant W \leqslant c_+^{(k)} \tag{5.3}$$

which can be called the shock adiabatic of the k-th shock wave. Comparison of (5.3) with the data of Fig.2 shows that the evolutionarity conditions are satisfied.

This shock adiabatic, constructed for discontinuities with $u_1^+ > u_1^\circ$ can be supplemented in a continuous manner within the domain $u_1 < u_1^\circ$ (where no change occurs in the elastic properties) by a line passing through the point u_i^- parallel to the k-th eigenvector of the matrix A_{ij}^-. This line corresponds to discontinuities within the domain V^*, where, as follows from the results in Sect.2, $W = c_-^{(k)}$ on this whole line. It can be shown that such a complete shock adiabatic will experience a break on intersecting the surface $u_1 = u_1^\circ$ and for $W \to \lambda_+^{(k)}$, the tangent to it will tend to coincide with the appropriate eigenvector in the domain V^{**} as $u_i \to \infty$.

Jumps from the surface of separation in the domain V^{**} will also be referred to the shocks. As follows from Sect.2 or directly from (5.2), these jumps occur in the direction of the eigenvector of the matrix A_{ij} and propagated at the characteristic velocity. There is no entropy change in such jumps.

Therefore all the fronts that are non-trivial low-intensity discontinuities in the neighbourhood of the surface of separation on which the conservation laws, the displacement continuity conditions, and the condition of a non-decrease in the entropy are satisfied are either jumps (taking account of the boundaries) in one of the domains V^* and V^{**} or jumps from V^* into V^{**} (shockwaves). All these discontinuities are evolutionary and no other non-trivial discontinuities satisfying the conditions mentioned exist.

6. *Continuous fronts, additional relationships.* A change' can obviously occur in the elastic properties of the medium as well as at the fronts with a continuous change in magnitudes. The conditions of continuity for u_i, v_i, S as well as the condition for crossing $\psi(u_i, S, x) = 0$, i.e., eight conditions, must be satisfied on these fronts. If only these conditions are satisfied, then for evolutionarity the front velocity (Fig.2) should lie in one of the following intervals:

$$0 < W < c_*^{(1)}, \quad c_{**}^{(1)} < W < c_*^{(2)}, \quad c_{**}^{(2)} < W < c_*^{(3)}, \quad c_{**}^{(3)} < W \tag{6.1}$$

Analogous continuous fronts for purely longitudinal waves were considered and used to contruct solutions in /9-11/, where they are called signotons.

As is seen from Fig.2, if the passage from V^{**} to V^* occurs in a continuous front and W satisfies one of the inequalities

$$c_*^{(1)} < W < c_{**}^{(1)}, \quad c_*^{(2)} < W < c_{**}^{(2)}, \quad c_*^{(3)} < W < c_{**}^{(3)} \tag{6.2}$$

then still another "additional" condition should be added at the front to the eight conditions listed above for evolutionarity of the discontinuity. This is related to the appearance of an additional characteristic leaving the front in the case of (6.2). All the families of characteristics for the fronts (6.1) pass on one side of the front or the other, while there is one family of characteristics for the fronts (6.2) that leave from the front on both sides. It can be said that perturbations associated with this family of characteristics are radiated by the front. Consequently, continuous fronts of the type (6.2) can be called radiating as opposed to the non-radiating fronts (6.1).

To set up the additional relationship we start from the following physical model of a discontinuity. We will assume that the change in elastic properties of a medium occurs in a continuous manner in a narrow layer in the space u_i, and the width of the layer then tends to zero. The equations of elasticity theory are assumed valid inside, as well as outside, such a layer. In this case the line representing a front of k-th type in the xt plane is replaced by a narrow strip within which a set of k-th family characteristics passes by, emerging from this cavity through both boundaries and leaving from it. The characteristic that was within this strip during this time can be found in any time interval. This means tht a relationship can be written in the k-th characteristic by taking into account that the velocity of its motion agrees, in the limit, with the velocity of motion of the front

$$|A_{ij} - \rho W^2 \delta_{ij}| = 0 \tag{6.3}$$
$$\rho g_i \left(\frac{\partial v_i}{\partial t} + W \frac{\partial v_i}{\partial x}\right) - \rho \dot{W} g_i \left(\frac{\partial u_i}{\partial t} + W \frac{\partial u_i}{\partial x}\right) = B_i g_i \frac{\partial S}{\partial x} + g_i C_i + g_i F_i$$

(g_i is an eigenvector of the matrix A_{ij}). The first of Eqs.(6.3) enables us to find an appropriate value of A_{11} for a given W (it can be seen that if $c_*^{(k)} < W < c_{**}^{(k)}$ then $A_{11}^* < A_{11} < A_{11}^{**}$). After this the second equation (6.3) yields the desired additional relationship at the front, which can be written in the form

$$\rho g_i \left(\frac{dv_i}{dt} - W \frac{du_i}{dt}\right) = B_i g_i \frac{\partial S}{\partial x} + g_i C_i + g_i F_i \tag{6.4}$$

(the direct derivatives denote differentiation along the front).

In general, there can also be other models of a continuous radiating front; however the modification considered possesses the attractive quality that it remains within the framework of elasticity theory.

In this connection, let us recall that fronts were considered /16/ on which a connection was made between the solutions of the wave equation and a first-order equation. Among the different kinds of fronts a radiating front ("a boundary of separation of the second kind") was also introduced. It is interesting to note that the additional condition on this front, obtained /16/ from all the other representations about its structure, agrees with the additional conditions found above that express the relationship on the characteristic.

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SINGULARITIES OF THE INTERACTION OF A VIBRATING STAMP WITH AN INHOMOGENEOUS HEAVY BASE*

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A method is developed for studying the fundamental characteristics of the wave process on the surface of an initally isotropic prestressed elastic half-space caused by an oscillating rigid stamp. The following is taken as the model of the inhomogeneous medium: an elastic layer $0 \leqslant x_3 \leqslant h,\ x_1, x_2 < \infty$ whose mechanical characteristics as well as the initial stresses are arbitrary, fairly smooth functions of the coordinate x_3 in the general case, lies on the surface of a homogeneous half-space $x_3 \geqslant h,\ x_1, x_2 < \infty$ (x_1, x_2, x_3 are a rectangular Cartesian coordinate system). The linearized boundary value problem of the dynamic theory of elasticity of vibrations with frequency ω for a rigid stamp on the surface of an inhomogeneous medium reduces to an integral equation or to

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